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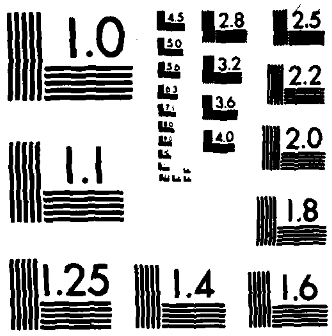

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

For a nonequilibrium relativistic beam-plasma system, an expression is obtained for the time-average force acting on a relativistic particle to second order in the total electromagnetic field. This relation is necessary for calculations of collective radiation effects and the conditions for the occurrence of radiative instability.

CONTENTS

1. INTRODUCTION

In the theory of electromagnetic radiation processes in nonequilibrium relativistic beam-plasma systems, an important role is played by the time-average nonlinear force acting on a relativistic test particle due to the total electromagnetic field. A major nonlinearity in the field arises from the dependence of the Lorentz force on the particle trajectory, which, in turn, depends on the field. Thereby the force has effectively a nonlinear dependence on the total electromagnetic field.

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In the present ~~work~~, the time-average of this nonlinear force is derived to second order in the total field for the case of unpolarized and slightly perturbed relativistic particle trajectories. The latter condition is that of applicability of the classical Born approximation for a plasma, namely, that the relativistic particle momentum be much greater than the electromagnetic impulse received by the particle in a time interval given by the inverse plasma frequency. Possible important screening effects arising from test particle polarization[1] are ignored here but will be fully addressed in another report.[*] The result of the present calculation agrees with that of Akopyan and Tsytovich.[1] It is important in calculations of collective radiation and the conditions for the occurrence of radiative instability in beam-plasma systems.

2. THE SECOND-ORDER NONLINEAR FORCE

A charged test particle in a beam-plasma experiences a force given by

$$\vec{F}_\alpha = e_\alpha\left[\vec{E}(\vec{r}_\alpha,t) + \vec{v}_\alpha \times \vec{B}(\vec{r}_\alpha,t)\right] + F_{dp} \quad . \tag{1}$$

Here α is a label designating the species of a particular particle, whose position and velocity at time t are given by $\vec{r}_\alpha$ and $\vec{v}_\alpha$, respectively. $\vec{E}(\vec{r}_\alpha,t)$ and $\vec{B}(\vec{r}_\alpha,t)$ are the electric and magnetic field, respectively, at the position of the particle at a particular time. The charge e_α is the actual charge of the particle. The quantity $\vec{F}_{dp}$, the dynamic polarization force, is a force correction arising from the polarization charge surrounding the test particle. The latter arises from the redistribution of other plasma particles due to the induced fields around the test particle. This polarization charge tends to screen the actual charge. In the present discussion the time-average of the polarization force is ignored; namely, it is assumed that conditions in the beam-plasma are such that

$$\langle\vec{F}_{dp}\rangle \approx 0 \quad . \tag{2}$$

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

**H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-82-6 (May 1982) to be published as HDL-TR-1994.*

It is however important to emphasize that in a complete analysis of collective radiation effects the dynamic polarization effect must be included. For the present discussion, substituting equation (2) into equation (1), then gives effectively

$$\vec{F}_\alpha = e_\alpha\left[\vec{E}(\vec{r}_\alpha, t) + \vec{v}_\alpha \times \vec{B}(\vec{r}_\alpha, t)\right] \quad . \tag{3}$$

The relativistic equation of motion governing the motion of the particle is given by

$$m_\alpha \frac{d}{dt}\left(\gamma_\alpha \vec{v}_\alpha\right) = \vec{F}_\alpha \quad , \tag{4}$$

where m_α is the rest mass of the particle and γ_α is the relativistic gamma given by

$$\gamma_\alpha = \left[1 - \left(\frac{v_\alpha}{c}\right)^2\right]^{-1/2} \quad . \tag{5}$$

The force $\vec{F}_\alpha$ may be written in terms of its Fourier transform $\vec{F}_{\alpha k}$; namely,

$$\vec{F}_\alpha = \int dk \vec{F}_{\alpha k}\, e^{-i\left(\omega t - \vec{k}\cdot\vec{r}_\alpha\right)} \quad , \tag{6}$$

where ω and $\vec{k}$ are the angular frequency and wave vector, respectively, and

$$dk \equiv d^3\vec{k}\, d\omega \quad . \tag{7}$$

Similarly, for the electromagnetic field Fourier decomposition, at the particle, one has

$$\vec{E} = \int dk \vec{E}_k\, e^{-i\left(\omega t - \vec{k}\cdot\vec{r}_\alpha\right)} \tag{8}$$

and

$$\vec{B} = \int dk \vec{B}_k\, e^{-i\left(\omega t - \vec{k}\cdot\vec{r}_\alpha\right)} \quad . \tag{9}$$

Substituting equations (6,8,9) in equation (3), then

$$\vec{F}_{\alpha k} = e_\alpha\left(\vec{E}_k + \vec{v}_\alpha \times \vec{B}_k\right) \quad . \tag{10}$$

Furthermore, from the Maxwell equation one has

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} \quad . \tag{11}$$

It follows that

$$\vec{B}_k = \frac{\vec{k} \times \vec{E}_k}{\omega + i\delta} \quad , \tag{12}$$

where a small imaginary part δ of the frequency is made explicit. Substituting equation (12) in equation (10), then

$$\vec{F}_{\alpha k} = e_\alpha \left[\vec{E}_k + \frac{\vec{v}_\alpha \times \left(\vec{k} \times \vec{E}_k\right)}{\omega + i\delta} \right] \quad . \tag{13}$$

Furthermore, by a well-known vector identity,

$$\vec{v}_\alpha \times \left(\vec{k} \times \vec{E}_k\right) = \left(\vec{v}_\alpha \cdot \vec{E}_k\right)\vec{k} - \left(\vec{v}_\alpha \cdot \vec{k}\right)\vec{E}_k \quad . \tag{14}$$

Substituting equation (14) in equation (13), then

$$\vec{F}_{\alpha k} = e_\alpha \left[\vec{E}_k \left(1 - \frac{\vec{v}_\alpha \cdot \vec{k}}{\omega + i\delta}\right) + \vec{k}\, \frac{\left(\vec{v}_\alpha \cdot \vec{E}_k\right)}{\omega + i\delta} \right] \quad . \tag{15}$$

Then substituting equation (15) in equation (6), the force on the particle to second order in the total field is expressed in terms of the particle trajectory and the Fourier component of the total electric field as follows:

$$\vec{F}_\alpha = e_\alpha \int dk \left[\vec{E}_k \left(1 - \frac{\vec{v}_\alpha \cdot \vec{k}}{\omega + i\delta}\right) + \vec{k}\, \frac{\left(\vec{v}_\alpha \cdot \vec{E}_k\right)}{\omega + i\delta} \right] e^{-i\left(\omega t - \vec{k} \cdot \vec{r}_\alpha\right)} \quad . \tag{16}$$

The time-average force acting on the actual charge of the particle is then given by

$$\langle \vec{F}_\alpha \rangle = \lim_{t \to \infty} \frac{1}{t} \int_{-t/2}^{t/2} \vec{F}_\alpha(t')dt' \quad . \tag{17}$$

Substituting equation (16) in equation (17) then

$$\langle \vec{F}_\alpha \rangle = \lim_{t \to \infty} \frac{e_\alpha}{t} \int_{-t/2}^{t/2} dt' dk \left[\vec{E}_k \left(1 - \frac{\vec{v}_\alpha \cdot \vec{k}}{\omega + i\delta}\right) + \vec{k} \left(\frac{\vec{v}_\alpha \cdot \vec{E}_k}{\omega + i\delta}\right) \right] e^{-i\left(\omega t' - \vec{k} \cdot \vec{r}_\alpha\right)} \quad . \tag{18}$$

This force depends on the particle trajectory through the particle position $\vec{r}_\alpha$ and velocity $\vec{v}_\alpha$ in equation (18). The classical Born approximation for a plasma is to be assumed here, namely, that a scattering of the particle by the field produces only a small perturbation of the particle trajectory in the plasma. Quantitatively, one requires that the particle momentum $\vec{p}_\alpha$ be much greater than the impulse received by the particle in a time interval given by the inverse plasma frequency ω_{pe}, namely,

$$\left|\vec{p}_\alpha\right| \gg e_\alpha\left|\vec{E}_\alpha\right|\left(\frac{1}{\omega_{pe}}\right) \quad . \tag{19}$$

Equation (4) is then to be solved for the motion by iteration to second order in

$$\frac{e_\alpha\left|\vec{E}_\alpha\right|}{\omega_{pe}\left|\vec{p}_\alpha\right|} \ll 1 \quad . \tag{20}$$

Substituting equation (16) in equation (4) one has

$$m_\alpha \frac{d}{dt}\left(\gamma_\alpha \vec{v}_\alpha\right) = e_\alpha \int dk \left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_\alpha}{\omega + i\delta}\right) + \vec{k}\left(\frac{\vec{v}_\alpha\cdot\vec{E}_k}{\omega + i\delta}\right)\right] e^{-i\left(\omega t - \vec{k}\cdot\vec{r}_\alpha\right)} \quad . \tag{21}$$

To lowest order, the motion of a particle taken to be located at the origin of coordinates at time t = 0 and moving with velocity $\vec{v}_\alpha$ is given by

$$\vec{r}_\alpha \approx \vec{v}_{\alpha o} t \tag{22}$$

and

$$\vec{v}_\alpha \approx \vec{v}_{\alpha o} \quad , \tag{23}$$

where $\vec{v}_{\alpha o}$ is a constant, namely, the unperturbed incident velocity. Substituting equations (22,23) in the right-hand side of equation (21) and integrating, then

$$\begin{aligned} m_\alpha \gamma_\alpha \vec{v}_\alpha = m_\alpha \gamma_{\alpha o} \vec{v}_{\alpha o} + e_\alpha \int dk \Bigg[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega + i\delta}\right) \\ + \vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega + i\delta}\right)\Bigg] \frac{e^{-i\left(\omega - \vec{k}\cdot\vec{v}_{\alpha o}\right)t}}{-i\left(\omega - \vec{k}\cdot\vec{v}_{\alpha o} + i\delta\right)} \quad , \end{aligned} \tag{24}$$

where

$$\gamma_{\alpha o} = \left[1 - \left(\frac{v_{\alpha o}}{c}\right)^2\right]^{-1/2} \quad . \tag{25}$$

and the motion is unperturbed in the distant past. Dividing both sides of equation (24) by $m_\alpha \gamma_\alpha$ then

$$\vec{v}_\alpha = \frac{\gamma_{\alpha o}\vec{v}_{\alpha o}}{\gamma_\alpha} + \frac{e_\alpha}{\gamma_\alpha m_\alpha}\int dk \left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega + i\delta}\right)\right.$$

$$\left. + \vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega + i\delta}\right)\right]\frac{e^{-i(\omega - \vec{k}\cdot\vec{v}_{\alpha o})t}}{-i(\omega - \vec{k}\cdot\vec{v}_{\alpha o} + i\delta)} \quad . \tag{26}$$

Using equation (5), one has that

$$\frac{\gamma_{\alpha o}}{\gamma_\alpha} = \gamma_{\alpha o}\left[1 - \left(\frac{v_\alpha}{c}\right)^2\right]^{1/2} \quad . \tag{27}$$

One defines the small perturbation $\Delta\vec{v}_\alpha$ in the particle velocity produced by the scattering process as follows:

$$\vec{v}_\alpha = \vec{v}_{\alpha o} + \Delta\vec{v}_\alpha \quad . \tag{28}$$

Substituting equation (28) in equation (27), then

$$\frac{\gamma_{\alpha o}}{\gamma_\alpha} = \gamma_{\alpha o}\left[1 - \left(\frac{\vec{v}_{\alpha o} + \Delta\vec{v}_\alpha}{c}\right)^2\right]^{1/2} \quad . \tag{29}$$

Expanding equation (29) to first order in $\Delta\vec{v}_\alpha$, then

$$\frac{\gamma_{\alpha o}}{\gamma_\alpha} = 1 - \gamma_{\alpha o}^2 \frac{\vec{v}_\alpha\cdot\Delta\vec{v}_a}{c^2} \quad . \tag{30}$$

Next, substituting equation (30) in equation (26), then

$$\vec{v}_\alpha = \vec{v}_{\alpha o} - \gamma_{\alpha o}^2\,\vec{v}_{\alpha o}\,\frac{\vec{v}_{\alpha o}\cdot\Delta\vec{v}_\alpha}{c^2} + \frac{e_\alpha}{\gamma_\alpha m_\alpha}\int dk \left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega + i\delta}\right)\right.$$

$$\left. + \vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega + i\delta}\right)\right]\frac{e^{-i(\omega - \vec{k}\cdot\vec{v}_{\alpha o})t}}{-i(\omega - \vec{k}\cdot\vec{v}_{\alpha o} + i\delta)} \quad . \tag{31}$$

Comparing equations (28) and (31), then

$$\Delta\vec{v}_\alpha = -\gamma_{\alpha o}^2\vec{v}_{\alpha o}\,\frac{\vec{v}_{\alpha o}\cdot\Delta\vec{v}_\alpha}{c^2}$$

$$+ \frac{e_\alpha}{\gamma_\alpha m_\alpha}\int dk\left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega + i\delta}\right) + \vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega + i\delta}\right)\right]\frac{e^{-i(\omega - \vec{k}\cdot\vec{v}_{\alpha o})t}}{-i(\omega - \vec{k}\cdot\vec{v}_{\alpha o} + i\delta)} \quad . \tag{32}$$

Next, dotting $\vec{v}_{\alpha o}$ into both sides of equation (32) and combining terms, then

$$\vec{v}_{\alpha o}\cdot\Delta\vec{v}_\alpha\left(1+\frac{\gamma_{\alpha o}^2\, v_{\alpha o}^2}{c^2}\right)=\frac{e_\alpha}{\gamma_\alpha m_\alpha}\int dk\,\vec{v}_{\alpha o}\cdot\left[\vec{E}_k\left(1-\frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega+i\delta}\right)+\vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega+i\delta}\right)\right]\frac{e^{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}\right)t}}{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}+i\delta\right)}\;. \tag{33}$$

Substituting equation (25) in the left-hand side of equation (33) and simplifying, then

$$\gamma_{\alpha o}^2\;\vec{v}_{\alpha o}\cdot\Delta\vec{v}_\alpha=\frac{e_\alpha}{\gamma_\alpha m_\alpha}\int dk\;\vec{v}_{\alpha o}\cdot\vec{E}_k\;\frac{e^{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}\right)t}}{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}+i\delta\right)}\;. \tag{34}$$

Next, substituting equation (34) in the second term on the right-hand side of equation (31), and to the required order replacing γ_α by $\gamma_{\alpha o}$, one has

$$\vec{v}_\alpha=\vec{v}_{\alpha o}+\frac{e_\alpha}{\gamma_{\alpha o}m_\alpha}\int dk\left[\vec{E}_k\left(1-\frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega+i\delta}\right)+\vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega+i\delta}\right)\right]\frac{e^{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}\right)t}}{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}+i\delta\right)}$$
$$-\frac{e_\alpha\vec{v}_{\alpha o}}{\gamma_{\alpha o}m_\alpha c^2}\int dk\;\vec{v}_{\alpha o}\cdot\vec{E}_k\frac{e^{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}\right)t}}{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}+i\delta\right)}\;. \tag{35}$$

Integrating equation (35) with constants of integration conforming to equation (22), and such that the motion is unperturbed in the distant past, then after simplifying one obtains

$$\vec{r}_\alpha=\vec{v}_{\alpha o}t-\frac{e_\alpha}{\gamma_{\alpha o}m_\alpha}\int dk\left[\vec{E}_k\left(1-\frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega+i\delta}\right)+\vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega+i\delta}\right)\right]\frac{e^{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}\right)t}}{\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}+i\delta\right)^2}$$
$$+\frac{e_\alpha}{\gamma_{\alpha o}m_\alpha c^2}\,\vec{v}_{\alpha o}\int dk\;\vec{v}_{\alpha o}\cdot\vec{E}_k\frac{e^{-i\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}\right)t}}{\left(\omega-\vec{k}\cdot\vec{v}_{\alpha o}+i\delta\right)^2}\;. \tag{36}$$

Equations (35) and (36) express the perturbed motion of the particle in the first Born approximation. It is this perturbed motion that enters in equation (18).

Substituting equations (35) and (36) in equation (18), then

$$
\begin{aligned}
\langle \vec{F}_\alpha \rangle = \lim_{t\to\infty} \frac{e_\alpha}{t} \int_{-t/2}^{t/2} dt'\,dk \Bigg\{ & \vec{E}_k - \frac{\vec{E}_k}{\omega + i\delta}\, \vec{k}\cdot\Bigg[\vec{v}_{\alpha o} \\
& + \frac{e_\alpha}{\gamma_{\alpha o} m_\alpha} \int dk_1 \left(\vec{E}_{k_1}\left(1 - \frac{\vec{k}_1\cdot\vec{v}_{\alpha o}}{\omega_1 + i\delta}\right) + \vec{k}_1\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{\omega_1 + i\delta}\right)\right) \frac{e^{-i(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o})t'}}{-i(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)} \\
& - \frac{e_\alpha \vec{v}_{\alpha o}}{\gamma_{\alpha o} m_\alpha c^2} \int dk_1\, \vec{v}_{\alpha o}\cdot\vec{E}_{k_1} \frac{e^{-i(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o})t'}}{-i(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)}\Bigg] + \frac{\vec{k}}{\omega + i\delta}\, \vec{E}_k\cdot\Bigg[\vec{v}_{\alpha o} \\
& + \frac{e_\alpha}{\gamma_{\alpha o} m_\alpha} \int dk_1 \left(\vec{E}_{k_1}\left(1 - \frac{\vec{k}_1\cdot\vec{v}_{\alpha o}}{\omega_1 + i\delta}\right) + \frac{\vec{k}_1}{\omega_1 + i\delta}(\vec{v}_{\alpha o}\cdot\vec{E}_{k1})\right) \frac{e^{-i(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o})t'}}{-i(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)} \\
& - \frac{e_\alpha \vec{v}_{\alpha o}}{\gamma_{\alpha o} m_\alpha c^2} \int dk_1 \vec{v}_{\alpha o}\cdot\vec{E}_{k_1} \frac{e^{-i(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o})t'}}{-i(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)}\Bigg]\Bigg\}\, e^{-i(\omega - \vec{k}\cdot\vec{v}_{\alpha}} \qquad (37) \\
& \times \Bigg[1 + \frac{ie_\alpha}{\gamma_{\alpha o} m_\alpha} \int dk_2 \left(\vec{k}\cdot\vec{E}_{k_2}\left(1 - \frac{\vec{k}_2\cdot\vec{v}_{\alpha o}}{\omega_2 + i\delta}\right) + \frac{\vec{k}\cdot\vec{k}_2(\vec{v}_{\alpha o}\cdot\vec{E}_{k_2})}{\omega_2 + i\delta}\right) \\
& \times \frac{e^{-i(\omega_2 - \vec{k}_2\cdot\vec{v}_{\alpha o})t'}}{-(\omega_2 - \vec{k}_2\cdot\vec{v}_{\alpha o} + i\delta)^2} - \frac{ie_\alpha}{\gamma_{\alpha o} m_\alpha c^2}\, \vec{k}\cdot\vec{v}_{\alpha o} \int dk_2 \frac{\vec{v}_{\alpha o}\cdot\vec{E}_{k_2} e^{-i(\omega_2 - \vec{k}_2\cdot\vec{v}_{\alpha o})t'}}{-(\omega_2 - \vec{k}_2\cdot\vec{v}_{\alpha o} + i\delta)^2}\Bigg] ,
\end{aligned}
$$

where the last exponential has been expanded to the required order. Combining terms to second order in accord with equation (20), and noting that the limits of time integration are infinite, then

$$\langle \vec{F}_\alpha \rangle = \lim_{t\to\infty} \frac{e_\alpha}{t} \int_{-\infty}^{\infty} dt' dk \left[\vec{E}_k - \vec{E}_k \frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega + i\delta} + \vec{k} \frac{\vec{E}_k\cdot\vec{v}_{\alpha o}}{\omega + i\delta} \right] e^{-i\left(\omega - \vec{k}\cdot\vec{v}_{\alpha o}\right)t'}$$

$$+ \lim_{t\to\infty} \frac{ie_\alpha^2}{t\gamma_{\alpha o} m_\alpha} \int_{-\infty}^{\infty} dt' \int dk dk_1 e^{-i\left[\left(\omega+\omega_1\right)-\left(\vec{k}+\vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right]t'}$$

$$\times \left\{ \frac{-\vec{E}_k \vec{k}\cdot\vec{E}_{k_1} \left(1 - \frac{\vec{k}_1\cdot\vec{v}_{\alpha o}}{\omega_1 + i\delta}\right) - \vec{E}_k \frac{\vec{k}\cdot\vec{k}_1}{\omega_1 + i\delta} \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{(\omega + i\delta)\left(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta\right)} \right.$$

$$+ \frac{\vec{E}_k \vec{k}\cdot\vec{v}_{\alpha o} \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{c^2(\omega + i\delta)\left(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta\right)} + \frac{\vec{k}\vec{E}_k\cdot\vec{E}_{k_1}\left(1 - \frac{\vec{k}_1\cdot\vec{v}_{\alpha o}}{\omega_1 + i\delta}\right) + \vec{k}\vec{E}_k\cdot\vec{k}_1 \frac{\vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{\omega_1 + i\delta}}{(\omega + i\delta)\left(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta\right)}$$

$$- \frac{\vec{k}\, \vec{E}_k\cdot\vec{v}_{\alpha o}\, \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{c^2(\omega + i\delta)\left(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta\right)} \tag{38}$$

$$+ \frac{\left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega + i\delta}\right) + \vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega + i\delta}\right)\right]\left[\vec{k}\cdot\vec{E}_{k_1}\left(1 - \frac{\vec{k}_1\cdot\vec{v}_{\alpha o}}{\omega_1 + i\delta}\right) + \vec{k}\cdot\vec{k}_1\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{\omega_1 + i\delta}\right)\right]}{-\left(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta\right)^2}$$

$$\left. + \frac{\left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega + i\delta}\right) + \vec{k}\left(\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega + i\delta}\right)\right] \vec{k}\cdot\vec{v}_{\alpha o}\, \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{c^2\left(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta\right)^2} \right\} .$$

Next, recalling the definition of the Dirac delta function, namely,

$$\delta(k) \equiv \delta^3\left(\vec{k}\right)\,\delta(\omega) = (2\pi)^{-4} \int d^3\vec{r} dt e^{-i\left(\vec{k}\cdot\vec{r} - \omega t\right)} , \tag{39}$$

and then performing the time integration in equation (38), one obtains

$$\langle F_\alpha\rangle = \lim_{t\to\infty} \frac{2\pi e_\alpha}{t} \int dk\,\delta(\omega - \vec{k}\cdot\vec{v}_{\alpha o}) \left[\vec{E}_k - \vec{E}_k \frac{\vec{k}\cdot\vec{v}_{\alpha o}}{\omega + i\delta} + \vec{k}\frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega + i\delta}\right]$$

$$+ \lim_{t\to\infty} \frac{2\pi i e_\alpha^{\,2}}{t\gamma_{\alpha o} m_\alpha} \int dk\,dk_1\,\delta(\omega + \omega_1' - (\vec{k} + \vec{k}_1)\cdot\vec{v}_{\alpha o}) \left[- \frac{\vec{E}_k \vec{k}\cdot\vec{E}_{k1}}{(\omega + i\delta)(\omega_1 + i\delta)}\right.$$

$$- \frac{\vec{E}_k \vec{k}\cdot\vec{k}_1 \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)} + \frac{\vec{E}_k \vec{k}\cdot\vec{v}_{\alpha o} \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{c^2\,(\omega + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)}$$

$$+ \frac{\vec{k}\ \vec{E}_k\cdot\vec{E}_{k_1}}{(\omega + i\delta)(\omega_1 + i\delta)} + \frac{\vec{k}\ \ \vec{E}_k\cdot\vec{k}_1\ \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)}$$

$$- \frac{\vec{k}\ \vec{E}_k\cdot\vec{v}_{\alpha o}\ \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{c^2\,(\omega + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)} - \frac{\vec{E}_k\ \vec{k}\cdot\vec{E}_{k_1}(\omega - \vec{k}\cdot\vec{v}_{\alpha o})}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)}$$

$$- \frac{\vec{E}_k\ \vec{k}\cdot\vec{k}_1\ \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}(\omega - \vec{k}\cdot\vec{v}_{\alpha o})}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)^2} \tag{40}$$

$$- \frac{\vec{k}\ \vec{v}_{\alpha o}\cdot\vec{E}_k\ \vec{k}\cdot\vec{E}_{k_1}}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)}$$

$$- \frac{\vec{k}\ \vec{v}_{\alpha o}\cdot\vec{E}_k\ \vec{k}\cdot\vec{k}_1\ \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)^2} + \frac{\vec{E}_k\vec{k}\cdot\vec{v}_{\alpha o}\ \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}(\omega - \vec{k}\cdot\vec{v}_{\alpha o})}{c^2\,(\omega + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)^2}$$

$$\left. + \frac{\vec{k}\ \vec{v}_{\alpha o}\cdot\vec{E}_k\ \vec{k}\cdot\vec{v}_{\alpha o}\ \vec{v}_{\alpha o}\cdot\vec{E}_{k_1}}{c^2(\omega + i\delta)(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} + i\delta)^2}\right] .$$

Next using the property of the delta function to replace $(\vec{k}\cdot\vec{v}_{\alpha o})$ by ω in the first integrand, and to replace $(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o})$ by $-(\omega - \vec{k}\cdot\vec{v}_{\alpha o})$ in the second integrand of equation (40), and simplifying, then the latter becomes

$$\langle\vec{F}_\alpha\rangle = \vec{F}_\alpha^{\,(1)} + \vec{F}_\alpha^{\,(2)} \quad , \tag{41}$$

where

$$\vec{F}_\alpha^{(1)} = \lim_{t\to\infty} \frac{2\pi}{t} e_\alpha \int dk\ \vec{k}\ \frac{\vec{v}_{\alpha o}\cdot\vec{E}_k}{\omega + i\delta}\ \delta\left(\omega - \vec{k}\cdot\vec{v}_{\alpha o}\right) \quad , \tag{42}$$

and

$$\vec{F}_\alpha^{(2)} = \lim_{t\to\infty} \frac{2\pi i}{t} e_\alpha$$

$$\times \int \frac{dk dk_1}{(\omega + i\delta)(\omega_1 + i\delta)}\ \delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\vec{k} E_{ki} E_{k_1 j} \Lambda_{ij}^{(\alpha)}(k_1,k) \quad , \tag{43}$$

where

$$\Lambda_{ij}^{(\alpha)}(k_1,k) =$$

$$\frac{e_\alpha}{\gamma_{\alpha o} m_\alpha}\left[\delta_{ij} + \frac{v_{\alpha o i}k_j - v_{\alpha o j}k_{1i}}{\omega - \vec{k}\cdot\vec{v}_{\alpha o} - i\delta} - \frac{v_{\alpha o i}v_{\alpha o j}}{\left(\omega - \vec{k}\cdot\vec{v}_{\alpha o} - i\delta\right)^2}\left(\vec{k}\cdot\vec{k}_1 - \frac{\omega\omega_1}{c^2}\right)\right] \quad . \tag{44}$$

Equation (43) can be symmetrized in k and k_1 as follows. Interchanging dummy integration variables $(\vec{k},\omega)$ with $(\vec{k}_1,\omega_1)$, and interchanging dummy indices i and j, then equation (43) becomes

$$\vec{F}_\alpha^{(2)} = \lim_{t\to\infty} \frac{2\pi i}{t} e_\alpha$$

$$\times \int \frac{dk dk_1}{(\omega_1 + i\delta)(\omega + i\delta)}\ \delta\left(\omega_1 + \omega - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\vec{k}_1 E_{k_1 j} E_{ki} \Lambda_{ji}^{(\alpha)}\left(k,k_1\right) \tag{45}$$

or, equivalently,

$$\vec{F}_\alpha^{(2)} = \lim_{t\to\infty} \frac{2\pi i}{t} e_\alpha$$

$$\times \int \frac{dk dk_1}{(\omega + i\delta)(\omega_1 + i\delta)}\ \delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\vec{k}_1 E_{ki} E_{k_1 j} \Lambda_{ji}^{(\alpha)}(k,k_1) \quad . \tag{46}$$

However, using equation (44) one has

$$\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\Lambda_{ji}^{(\alpha)}(k,k_1) = \frac{e_\alpha}{\gamma_{\alpha o} m_\alpha}\ \delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)$$

$$\times\left[\delta_{ji} + \frac{v_{\alpha o j}k_{1i} - v_{\alpha o i}k_j}{\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} - i\delta} - \frac{v_{\alpha o j}v_{\alpha o i}}{\left(\omega_1 - \vec{k}_1\cdot\vec{v}_{\alpha o} - i\delta\right)^2}\left(\vec{k}_1\cdot\vec{k} - \frac{\omega_1\omega}{c^2}\right)\right] \quad . \tag{47}$$

Using the properties of the Kronecker delta and the Dirac delta function, equation (47) may be rewritten as

$$\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\Lambda_{ji}^{(\alpha)}(k,k_1) = \frac{e_\alpha}{\gamma_{\alpha o} m_\alpha}\,\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)$$
$$\times\left[\delta_{ij} + \frac{v_{\alpha oi}k_j - v_{\alpha oj}k_{1i}}{\omega - \vec{k}\cdot\vec{v}_{\alpha o} + i\delta} - \frac{v_{\alpha oi}v_{\alpha oj}\left(\vec{k}\cdot\vec{k}_1 - \frac{\omega\omega_1}{c^2}\right)}{\left(\omega - \vec{k}\cdot\vec{v}_{\alpha o} + i\delta\right)^2}\right] . \tag{48}$$

Using equation (44) in equation (48) then the following symmetry relation is obtained:

$$\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\Lambda_{ji}^{(\alpha)}(k,k_1) = \delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\Lambda_{ij}^{(\alpha)*}(k_1,k) . \tag{49}$$

Substituting equation (49) in equation (46) then

$$F_\alpha^{(2)} = \lim_{t\to\infty}\frac{2\pi i}{t}\,e_\alpha\int\frac{dk\,dk_1}{(\omega + i\delta)(\omega_1 + i\delta)}$$
$$\times\ \delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\vec{k}_1 E_{ki}E_{k_1j}\Lambda_{ij}^{(\alpha)*}(k_1,k) . \tag{50}$$

Therefore $F_\alpha^{(2)}$ can be expressed as half the sum of the two equivalent expressions--equations (43) and (50)--namely,

$$F_\alpha^{(2)} = \lim_{t\to\infty}\frac{\pi i}{t}\,e_\alpha\int dk\,dk_1\,\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_{\alpha o}\right)\frac{1}{(\omega + i\delta)(\omega_1 + i\delta)}E_{ki}E_{k_1j}$$
$$\times\left[\vec{k}\,\Lambda_{ij}^{(\alpha)}(k_1,k) + \vec{k}_1\Lambda_{ij}^{(\alpha)*}(k_1,k)\right] . \tag{51}$$

Equations (42) and (51) agree with equations (7) and (8) of Akopyan and Tsytovich.[1] It is to be understood that $\vec{v}_\alpha$ and γ_α in that reference are unperturbed values which are here designated by $\vec{v}_{\alpha o}$ and $\gamma_{\alpha o}$, respectively. Also, since single-wave particle resonance is ignored there, the principal part of $\Lambda_{ij}^{(\alpha)*}(k_1,k)$ in equation (51) is understood, in which case $\Lambda_{ij}^{(\alpha)}$ may be treated as effectively real and becomes a common factor. The disparity of a factor of $(2\pi)^{-3}$ and $(2\pi)^{-6}$ in equations (7) and (8), respectively, of Akopyan and Tsytovich[1] results merely from the different Fourier transform convention chosen there. For example, in equation (5) of Akopyan and Tsytovich[1] the Fourier transform convention employed there has a factor of $(2\pi)^{-3}$ in the inverse Fourier transform in the integration over the three-dimensional wave

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

vector space, and a factor of 1 for the integration over frequency, giving a total factor of $(2\pi)^{-3}$, whereas here a total factor of 1 is used in equation (21). Therefore, the appropriate factor for the Fourier transform itself is $(2\pi)^{-1}$ there and $(2\pi)^{-4}$ here. Equations (41), (42), and (51) give the time-average nonlinear force to second order in the total field for slightly perturbed relativistic orbits.

3. CONCLUSION

An expression--equations (41), (42), and (51)--has been obtained for the time-average nonlinear force acting on an unpolarized relativistic test particle in a nonequilibrium beam-plasma system. This expression holds to second order in the total field under the condition that the classical Born approximation applies for the plasma. This result has been used in the work of Akopyan and Tsytovich in the theory of collective bremsstrahlung in nonequilibrium plasmas.

The present work, together with related work by the author[2-7,*,†] is important for ongoing work in the calculation of collective radiation processes and conditions for the occurrence of radiative instability in beam-plasma systems.

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[2]*H. E. Brandt, Symmetries of the Nonlinear Conductivity for a Relativistic Turbulent Plasma, Harry Diamond Laboratories, HDL-TR-1927 (March 1981).*

[3]*H. E. Brandt, Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma, Phys. Fluids, 24 (1981), 1760.*

[4]*H. E. Brandt, Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma, in Plasma Astrophysics, Course and Workshop organized by the International School of Plasma Physics, 27 August to 7 September 1981, Varenna (Como), Italy (European Space Agency ESA SP-161, Nov. 1981). (Also to be published by Pergamon Press, 361.)*

[5]*H. E. Brandt, On the Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma, Harry Diamond Laboratories, HDL-TR-1970 (February 1982).*

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[7]*H. E. Brandt, Symmetry of the Complete Second-Order Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma, J. Math. Phys. 24, (1983), 1332.*

**Some of the author's related work will be published in 1983 as Harry Diamond Laboratories reports: HDL-TR-1994, HDL-TR-1996, and HDL-TR-2009.*

†Other related work prepared in preprint form will be published later and is available from the author.

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